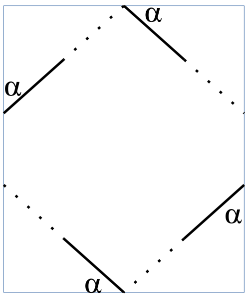
Making polyhedra with playing cards

You're probably reading this because you've seen some nice-looking playing card constructions, and you've thought – how hard is that to do? Can I do it?

The answers are "not very hard at all" and "yes".

There's one that you can start with straight away: you can build a tetrahedron (4-faced pyramid, every face is a triangle) very simply – take 6 cards and cut each of them like so



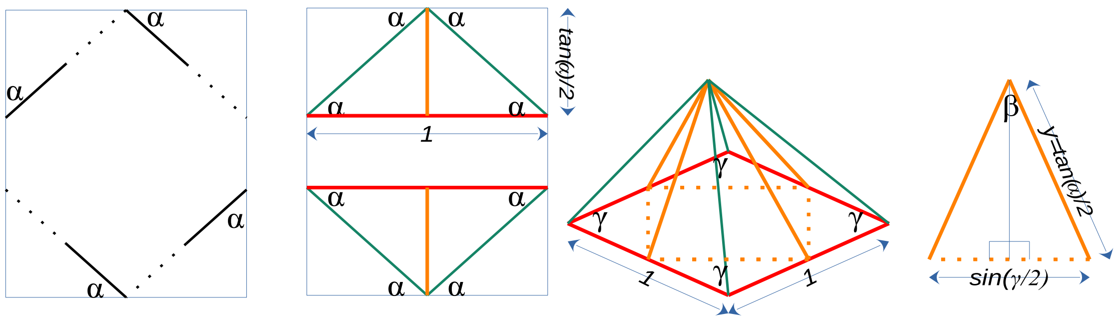
Where "α" (the angle between the cut line and the short edge of the card) is 45 degrees.

Slot them together, joining the cards so three come together at each short edge midpoint. Hard to describe in words, but not too tricky to do once you've fiddled for a bit.

The way this works is that each card represents an edge of the polyhedron. A tetrahedron has 6 edges, so you need 6 cards. What you need to do is figure out what value to use for "α" for each platonic solid. The next section shows you how the maths works – but you don't have to understand that; you can just get the values from the table at the bottom.

Figuring out cut angles

We assume a common template with cuts at each end as per the diagram (thick black lines are where you cut). Let's assume the playing card width is 1 (the red line below).We will be locking cards together to form a pyramid as below (the diagram shows 4 cards meeting – octahedron – but it could be 5 – icosahedron – or 3 – the other solids).



alpha is the cut angle against the short side. Red line's length of 1 => yellow line length is $y = 0.5 * \tan(alpha)$

beta is the angle between the yellow lines – these run along the main axis of the cards. This needs to match the platonic solid internal face angle, set by the number of sides **m** of each of the faces of the solid: $\beta = 180 - (360/m)$

gamma is the internal angle of the pyramid base. The number of base sides is the number of edges at each vertex (the number of cards coming together). So where we have **n** edges at each polygon vertex, $\gamma = 180 - (360/n)$

For a given Platonic solid, we know the number of sides of the faces and the number of edges at each vertex, so we know what beta and gamma are (see the table below). We want to determine what this implies for the value of alpha. To do this we need to relate thelength of the yellow line (which is a function of alpha) to beta and gamma.

We start by noting that beta is at the apex of an isoceles triangle: the base of the triangle is a line running between midpoints of neighbouring sides of the base of the pyramid, shown above by a dotted yellow line. The length of this dotted line is $\sin(\gamma/2)$ (you can figure this out from the pyramid's red base edge lines having length 1).

We can drop a perpendicular from the apex of the isoceles triangle to get a right-angled triangle where the side opposite the half-beta angle is half the length of the dotted line. Since the hypotenuse of the triangle is y, we know this side's length is also equal to $y * \sin(\beta/2)$. So $y * \sin(\beta/2) = 0.5 * \sin(\gamma/2)$.

Substituting for y gives us an expression relating all 3 angles $0.5 * \tan(\alpha) * \sin(\beta/2) = 0.5 * \sin(\gamma/2)$

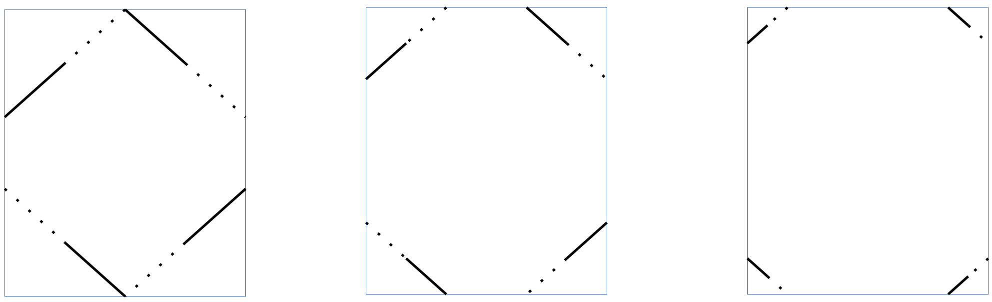
And rewriting to solve for alpha gives $\alpha = \arctan(\sin(\gamma/2)/\sin(\beta/2))$

Shape	Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
#Sides of face	3	4	3	5	3
beta	60	90	60	108	60
Lines at vertex (Pyramid base sides)	3	3	4	3	5
gamma	60	60	90	60	108
alpha	45	~ 35.2644	~ 54.7356	~ 31.7175	~ 58.2825
tan(alpha)	1	1/√2 ~ 0.707107	√2 ~ 1.41421	1/Phi ~0.618034	Phi ~1.618034
#Faces	4	6	8	12	20
#Vertexes	4	8	6	20	12
#Edges (#cards)	6	12	12	30	30

Building

One thing worth pointing out before you build the other solids is that you don't have to have your cut lines meeting in the middle of the short edge of the card: you can move them towards the edges and you'll still get cards meeting at the same angle.

This can be useful if your "alpha" angle is large (e.g. for octahedron and icosahedron in particular. So the below cut lines all have the same alpha and so will all form the same angles:



You will almost certainly need to do this for the icosahedron, as you won't be able to build it with standard poker cards otherwise.

Going Further

Another thing to note is that the formula for alpha will work for any beta and gamma value – it's not dependent on our being part of a platonic solid. It's just that platonic solids are the easiest place to start, because all the faces are the same.